Text Properties and Languages

Statistical Properties of Text

- How is the frequency of different words distributed?
- How fast does vocabulary size grow with the size of a corpus?
- Such factors affect the performance of information retrieval and can be used to select appropriate term weights and other aspects of an IR system.

Word Frequency

- A few words are very common.
  - 2 most frequent words (e.g. "the", "of") can account for about 10% of word occurrences.
- Most words are very rare.
  - Half the words in a corpus appear only once, called *hapax legomena* (Greek for "read only once")
- Called a "heavy tailed" or "long tailed" distribution, since most of the probability mass is in the "tail" compared to an exponential distribution.

Sample Word Frequency Data
(from B. Croft, UMass)

Frequent Word	Number of Occurrences	Percentage of Total
the	7,398,934	5.9
of	3,893,790	3.1
to	3,364,653	2.7
and	3,320,687	2.6
in	2,311,785	1.8
is	1,559,147	1.2
for	1,313,561	1.0
The	1,144,860	0.9
that	1,066,503	0.8
said	1,027,713	0.8

Frequencies from 336,310 documents in the 1GB TREC Volume 3 Corpus
125,720,891 total word occurrences; 508,209 unique words

Zipf's Law

- Rank (r): The numerical position of a word in a list sorted by decreasing frequency (f).
- Zipf (1949) "discovered" that:

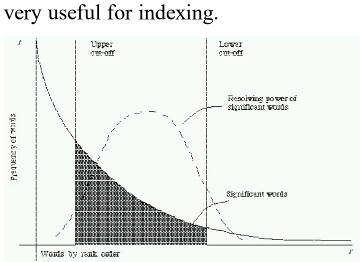
$$f \propto \frac{1}{r} \qquad f \cdot r = k \ \text{(for constant } k\text{)}$$

- If probability of word of rank r is p_r and N is the total number of word occurrences:

$$p_r = \frac{f}{N} = \frac{A}{r} \quad \text{for corpus indp. const. } A \approx 0.1$$

Zipf and Term Weighting

- Luhn (1958) suggested that both extremely common and extremely uncommon words were not very useful for indexing.



Prevalence of Zipfian Laws

- Many items exhibit a Zipfian distribution.
  - Population of cities
  - Wealth of individuals
    - Discovered by sociologist/economist Pareto in 1909
  - Popularity of books, movies, music, web-pages, etc.
  - Popularity of consumer products
    - Chris Anderson's "long tail"

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Predicting Occurrence Frequencies

- By Zipf, a word appearing n times has rank $r_n = AN/n$
- Several words may occur n times, assume rank r_n applies to the last of these.
- Therefore, r_n words occur n or more times and r_{n+1} words occur $n+1$ or more times.
- So, the number of words appearing **exactly** n times is:

$$I_n = r_n - r_{n+1} = \frac{AN}{n} - \frac{AN}{n+1} = \frac{AN}{n(n+1)}$$

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Predicting Word Frequencies (cont)

- Assume highest ranking term occurs once and therefore has rank $D = AN/1$
- Fraction of words with frequency n is:

$$\frac{I_n}{D} = \frac{1}{n(n+1)}$$

- Fraction of words appearing only once is therefore ½.

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Occurrence Frequency Data

(from B. Croft, UMass)

Number of Occurrences (n)	Predicted Proportion of Occurrences 1/n(n+1)	Actual Proportion occurring n times I_n/D	Actual Number of Words occurring n times
1	.500	.402	204,357
2	.167	.132	67,082
3	.083	.069	35,083
4	.050	.046	23,271
5	.033	.032	16,332
6	.024	.024	12,421
7	.018	.019	9,766
8	.014	.016	8,200
9	.011	.014	6,907
10	.009	.012	5,893

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Does Real Data Fit Zipf's Law?

- A law of the form $y = kx^c$ is called a power law.
- Zipf's law is a power law with $c = -1$
- On a log-log plot, power laws give a straight line with slope c.

$$\log(y) = \log(kx^c) = \log k + c\log(x)$$

- Zipf is quite accurate except for very high and low rank.

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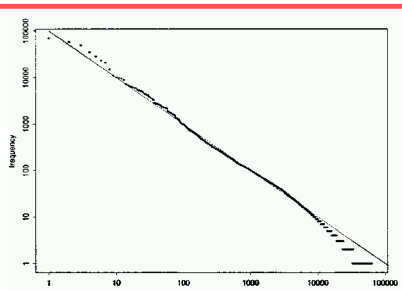
Fit to Zipf for Brown Corpus



$k = 100{,}000$

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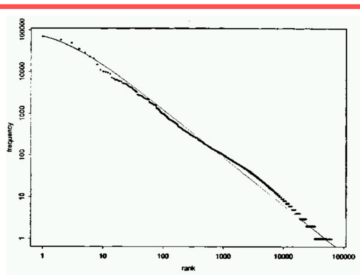
Mandelbrot (1954) Correction

- The following more general form gives a bit better fit:

$$f = P(r+\rho)^{-B} \qquad \text{For constants } P, B, \rho$$

Mandelbrot Fit



Mandelbrot's function on Brown corpus
$P = 10^{5.4}$, $B = 1.15$, $\rho = 100$

Explanations for Zipf's Law

- Zipf's explanation was his "principle of least effort." Balance between speaker's desire for a small vocabulary and hearer's desire for a large one.
- Debate (1955-61) between Mandelbrot and H. Simon over explanation.
- Simon explanation is "rich get richer."
- Li (1992) shows that just random typing of letters including a space will generate "words" with a Zipfian distribution.
  - http://linkage.rockefeller.edu/wli/zipf/

Zipf's Law Impact on IR

- Good News:
  - Stopwords will account for a large fraction of text so eliminating them greatly reduces inverted-index storage costs.
  - Postings list for most remaining words in the inverted index will be short since they are rare, making retrieval fast.
- Bad News:
  - For most words, gathering sufficient data for meaningful statistical analysis (e.g. for correlation analysis for query expansion) is difficult since they are extremely rare.

Vocabulary Growth

- How does the size of the overall vocabulary (number of unique words) grow with the size of the corpus?
- This determines how the size of the inverted index will scale with the size of the corpus.
- Vocabulary not really upper-bounded due to proper names, typos, etc.

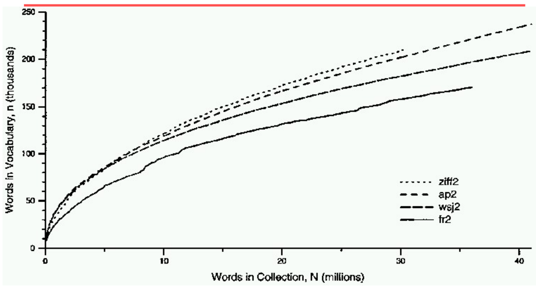
Heaps' Law

- If V is the size of the vocabulary and the n is the length of the corpus in words:

$$V = Kn^{\beta} \quad \text{with constants } K,\ 0 < \beta < 1$$

- Typical constants:
  - $K \approx 10–100$
  - $\beta \approx 0.4–0.6$ (approx. square-root)

Heaps' Law Data



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Explanation for Heaps' Law

- Can be derived from Zipf's law by assuming documents are generated by randomly sampling words from a Zipfian distribution.

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